II. Solution by E. B. ESCOTT, Ann Arbor, Michigan.

The solution of the congruence $x^2+y^2\equiv 0 \pmod 5$, is $x\equiv \pm 2y \pmod 5$.
Let $x=5a\pm 2y$, and substitute in $x^2+y^2\equiv 0 \pmod{5^2}$.
We get $x\equiv \pm 7y \pmod{5^3}$, and proceeding in this way we get finally,

$$x\equiv \pm 182y \pmod{5^4}.$$

NOTE. A similar and more complete solution by the Proposer has already been published. ED. W.

180. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values of x and y such that $96x-96y+21=\square$.

II. Solution by B. KRAMER, E. M., Pittsburg, Pennsylvania.

Let $x-y=z$. Then $96z+21=u^2$. From this it is easily seen that u is odd, and $u^2\equiv 5 \pmod 8$.

But if we put $u=2k+1$, $u^2=4k(k+1)+1$, which, since k or $k+1$ is even, says that $u^2\equiv 1 \pmod 8$. Hence no square can be congruent to 5 modulo 8, and the relation is impossible.

181. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

If $2n+1$ is an odd prime p, $(2n)!\equiv(-1)^n 2^{4n}(n!)^2 \pmod{p^2}$.

Solution by S. LEFSCHETZ, Clark University.

The given congruence can be written

$$\begin{aligned} 2n! &\equiv (-1)^n 2^{2n}[2.4\ldots 2n]^2 \pmod{p^2}, \text{ or} \\ 1.3.5\ldots(2n-1) &\equiv (-1)^n 2^{2n}.2.4.6\ldots 2n \pmod{p^2} \\ &\equiv (-1)^n 2^{2n}(p-1)(p-3)\ldots[p-(2n-1)] \\ \equiv 2^{2n}&[1.3\ldots(2n-1)-p.1.2\ldots(2n-1)(\tfrac{1}{1}+\tfrac{1}{3}+\ldots+\frac{1}{2n-1})] \pmod{p^2}. \end{aligned}$$

If we adopt Bachmann's notation [*Niedere Zahlentheorie*, Bd. 1, p. 161] and call $1/m$ the number m' such that $mm'\equiv 1 \pmod p$. We have to prove, as can be seen at once that:

$$\frac{2^{2n}-1}{p}\equiv \tfrac{1}{1}+\tfrac{1}{3}+\ldots+\frac{1}{2n-1} \pmod p,$$

and this is proved by Bachmann (loc. cit., p. 164).

182. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Find two general solutions in integers of the equation $x^2=616318177y-1$.

No solution of this problem has been received.